that decrease in volume is also represented by positive volumetric strain ($d\epsilon_v = -dv/v$, analogous to $\epsilon = -dl/l$). Thus we can take as the measure of the resistance to deformation the parameter

$$\sigma_w = \frac{dW}{d\epsilon_1} = \sigma + p\frac{d\epsilon_v}{d\epsilon_1}. \quad (1)\dagger$$

When the specimen is compacting, σ_w is higher than σ and when the specimen is dilating it is lower. The deformation itself is fully specified by ϵ_1 and ϵ_v but, when comparing behaviour in closely related triaxial tests, the plot of σ_w against ϵ_1 alone will often serve to characterize the stress–strain properties sufficiently for our purposes, especially when ϵ_1 is large compared with ϵ_v.

Two properties of the stress–strain behaviour that may be important in suggesting the nature of the deformation mechanism can now be expressed.

1. Work hardening. Defining work hardening as the change in resistance to deformation as the deformation proceeds, it can be derived from the triaxial test as

$$\frac{d\sigma_w}{d\epsilon_1} = \frac{d\sigma}{d\epsilon_1} + p\frac{d^2\epsilon_v}{d{\epsilon_1}^2}. \quad (2)$$

When the volume changes linearly with strain, $d\sigma_w/d\epsilon_1$ is the same as $d\sigma/d\epsilon_1$, the apparent rate of work hardening from the σ vs ϵ_1 plot, but curvature in the volume change vs strain plot introduces an additional work-hardening component, as recognized by FRANK [18] and BRACE *et al.* [2]. Thus, the effective work-hardening rate is greater than $d\sigma/d\epsilon_1$ when the ϵ_v vs ϵ_1 plot is concave towards positive ϵ_v, that is, when $\Delta v/v_0$ vs ϵ_1 (as plotted here) is concave downwards, and it is less than $d\sigma/d\epsilon_1$ when $\Delta v/v_0$ vs ϵ_1 is concave upwards. Put in another way, even if the intrinsic resistance to deformation does not change with strain, a changing porosity during a triaxial test gives rise to an apparent work hardening in the σ vs ϵ_1 plot when $\Delta v/v_0$ vs ϵ_1 is concave upwards or an apparent work softening when $\Delta v/v_0$ vs ϵ_1 is concave downwards.

2. Pressure sensitivity. The influence of pressure on the intrinsic resistance to deformation at any particular strain ϵ_1 is correspondingly represented by

$$\left(\frac{d\sigma_w}{dp}\right)_{\epsilon_1} = \left(\frac{d\sigma}{dp}\right)_{\epsilon_1} + \frac{d}{dp}\left(p\frac{d\epsilon_v}{d\epsilon_1}\right)_{\epsilon_1} \quad (3)$$

or

$$\tan\psi_w = \tan\psi + \frac{d}{dp}\left(p\frac{d\epsilon_v}{d\epsilon_1}\right)$$

where $\tan\psi$ is the slope of the $\sigma_{\epsilon,1}$ vs p curve, related to the slope $\tan\phi$ of the Mohr envelope for the stress state at given strain by [19]‡

$$\tan\phi = \frac{\tan\psi}{2\sqrt{(1+\tan\psi)}}.$$

† The corresponding expression for the shear test is

$$\frac{dW}{d\gamma} = \tau + \sigma_n\frac{d\epsilon_v}{d\gamma}$$

where $d\gamma$ is the increment in shear strain, and τ and σ_n are the shear stress and normal stress, respectively, on the shear plane.

‡ The use of $\tan\psi$ or $\tan\phi$ as an index of pressure sensitivity, as in this earlier paper, is only meaningful in the absence of significant volume changes.

Curves of σ_w^* vs ϵ_1 for the six materials are given in Figs 3(b)–8(b). The addition of the asterisk superscript to σ_w indicates that in deriving these from the observations given in Figs 3(a), (c)–8(a), (c), respectively, the elastic volume changes due to the applied differential stresses were first subtracted from the measured $\Delta v/v_0$ vs ϵ_1 curves; that is, in applying equation (1) we have used $d\epsilon_{vp}/d\epsilon_1$ instead of $d\epsilon_v/d\epsilon_1$ where ϵ_{vp} is the non-elastic part of the volumetric strain. This ensures that the $p(d\epsilon_{vp}/d\epsilon_1)$ term reflects only the work associated with non-elastic volume changes and does not include elastic work which is recovered when the differential stress is removed. In deriving the elastic volumetric strains from the measured differential stresses, the elastic moduli used, which for isotropic material should be equal to one-third of the compressibility, have been chosen to be approximately consistent with the volume changes measured when the differential stress is released at the end of the straining, as indicated by the vertical lines at the ends of the curves of Figs 3(c)–8(c). These moduli are larger than correspond to the compliances of the mineral grains themselves since they take into account elastic changes in pore volume as well. The values used were $0{\cdot}0007\ \text{kb}^{-1}$ for limestone, marble and talc, $0{\cdot}001\ \text{kb}^{-1}$ for sandstone and $0{\cdot}0014\ \text{kb}^{-1}$ for sodium chloride, but, as the examples in Figs 3(c)–8(c) (dotted lines) show, the elastic corrections are generally not so large relative to the total volume changes as to make the choice of these values particularly critical. In the case of graphite where a single value for all experiments is no longer a sufficient approximation because of the effect of large changes of porosity from one pressure to another and even during one experiment, an interpolated set of moduli was used, given in Fig. 7(e) as a function of instantaneous porosity and ranging from $0{\cdot}007$ to $0{\cdot}002\ \text{kb}^{-1}$. Another approximation affecting the calculation of σ_w^* is the use of conventional strains for both ϵ_1 and ϵ_v where, strictly, logarithmic strains should have been used, but within the limits of accuracy set by the data no serious error should arise from this.

It must be emphasized that the σ_w^* vs ϵ_1 curves in Figs 3(b)–8(b) are of a rather imprecise nature, especially at low strains†, in view of the limited accuracy of the data and its effect on the measurement of slopes and of the other approximations mentioned above. Nevertheless, it is believed that the σ_w^* vs ϵ_1 curves are a much better guide to the processes occurring in the specimen than are the σ vs ϵ_1 curves. We shall therefore now discuss the main features of the behaviour of the individual materials with reference to the σ_w^* vs ϵ_1 curves.

Limestone and marble [Figs 3(b) and 4(b)]. The differences in behaviour of the two calcite rocks suggested by the σ vs ϵ_1 curves appear less marked in several respects when the interaction of the volume changes is allowed for. Thus, the limestone curves no longer cross over at low strains, supporting the earlier suggestion [11] that this effect on the σ vs ϵ_1 is associated with an increased contribution of pore collapse during straining at higher pressures. Also, the effective work hardening of the marble now appears comparable to or even rather higher than that of the limestone; the high slopes of the σ vs ϵ_1 curves for the limestone are therefore to be attributed to a large $p(d^2\epsilon_{vp}/d\epsilon_1^2)$ effect rather than to an intrinsically high work-hardening rate associated with the mechanism of deformation. However, the limestone still appears much the stronger; its σ_w^* vs ϵ_1 curves in compression are generally at least 1 kb higher than those of the marble up to 4 kb confining pressure, with an increasing difference at higher pressures due to a higher pressure sensitivity of the limestone curves at the higher pressures.

† Little or no significance should be attached to the σ_w^* vs ϵ_1 curves below about 1 per cent strain. Note that, in principle, they need not pass through the origin, the case of graphite being a possible example of this.